



Demographic Shock and Social Security: A Political Economy Perspective

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Abstract

We assume that individual voters differ not only according to age but also productivity. In the steady state, workers with wages in the intermediate range join the retired persons to form a majority and vote for a positive level of social security. When a shock decreases population growth, entrenched interests can constrain majority voting decisions and prevent reforms in the name of entitlements. We show that from a Rawlsian viewpoint it may be desirable to rely on these entitlements to protect the low wage earners of the transition generations. However, when the possibility of fixing a basic pension is introduced, it constitutes a better instrument than entitlements.

Keywords: social security, majority voting, entitlements, aging

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1. Introduction

The aging of the population constitutes nowadays one of the most important economic problems faced by our societies. Most pension systems in the developed countries are of the Pay-As-You-Go (PAYG) type. This means that income is redistributed from the workers to the retired individuals. Because of falling mortality and fertility rates, the ratio of contributors to beneficiaries is decreasing. Consequently, governments must find solutions to ensure the financial viability of the PAYG systems. This means raising the retirement age, decreasing the replacement rate or increasing the rate of contribution to the system.

We argue elsewhere (Cremer and Pestieau, 2000) that the main problem facing our pension systems is not of a financial but of a political nature. The old age crisis could be averted through appropriate reforms. In particular, the key parameters of social security systems, namely the replacement ratio, the payroll taxes and the retirement age would have to be adjusted in a suitable way. Furthermore, individual or collective saving for retirement ought to be fostered. The real issue is thus a political one. Reforms are made difficult because of the political resistance of the various groups that compose society. In particular, any attempt to decrease the replacement rate is blocked by the retired individuals. It is thus necessary to

understand how the conflicts of interest, not only between old and young, but also between rich and poor, are resolved through the political process.

The literature dealing with the vote over pensions starts with the seminal paper of Browning (1975). This paper addresses the question of the majority voting equilibrium tax rate when the economy is in the *steady state*. Browning shows that preferred tax rates are increasing with age and therefore that the decisive voter belongs to the median generation. The tax rate chosen by the majority is then higher than the socially optimal tax rate, which corresponds to the preferred tax rate of the young. Several extensions of the Browning's model have been considered. Examples include Boadway and Wildasin (1989) who introduce (imperfect) capital markets, Hu (1982) who considers uncertainty about future pension benefits and Tabellini (2000) who introduced income heterogeneity among the workers.

None of these papers addresses the question of the political equilibrium out of steady state, more specifically when the rate of population growth is changing over time. Boldrin and Rustichini (2000) show that when the rate of population growth is subject to negative random shocks, the PAYG system gradually disappears. In the first periods, the rate of population growth being higher than the interest rate, the young, who constitute the majority, support the PAYG system. Then, when the rate of population growth becomes lower than the interest rate, the young vote for the abandonment of the system. The people young during the period just preceding the collapse of the PAYG system continue to vote for a positive tax rate because there is a positive probability that the system continues.

In this paper, we also characterize the evolution of the PAYG system chosen by majority voting when a shock on the rate of population growth occurs, although in a different setting. People live for two periods and are differentiated not only according to age but also to their income levels. Pension systems redistribute income between workers. They range from purely redistributive (Beveridgean) to purely contributive (Bismarckian).

We first recall some steady state results drawn from an earlier paper (Casamatta et al., 2000b). A positive tax rate may be chosen by the majority, even when the interest rate is larger than the rate of population growth (which represents the rate of return of the PAYG pension system).

We then turn to the case of an unexpected negative demographic shock. Comparative statics with respect to n , the rate of population growth, are ambiguous. However, one can expect that if the decline in n is large enough, the young who hold the majority will reject the PAYG system. Consequently, the voting equilibrium now implies no taxes and no social security benefits.

In that case, the retired persons in the transition period are left with few or even no resources. We argue that the retired individuals may be able to block the decision of the majority in the name of *entitlements*. We assume that they are able to obtain the payment of at least the Bismarckian, or contributive, part of their pension. The transition to the new steady state is then gradual; this new steady state implies the abandonment of the system. As an alternative, we also consider the possibility of a minimum pension, guaranteed to all the retired individuals. The immediate implication of this assumption is that the system never disappears, even when the poorest of the workers would like to abandon it.

Having characterized the dynamic trajectory of the pension system subsequent to a shock on the rate of population growth, we then study the determination of the optimal system, at an earlier, *constitutional stage*. Optimality is defined according to the Rawlsian criterion. We

show that some Bismarckian features, which imply the presence of entitlements, may be an effective way of protecting the transition generation against the “tyranny” of the majority. This is because majority voting does not lead to the optimal adjustments of retirement benefits in case of a demographic shock. Specifically, transition may be too sudden so that the burden of transition falls on a single generation. Entrenched interests constitute a mechanism that can mitigate this problem by providing the retired persons of the transition periods with a decent standard of living. However, when a basic pension is an available instrument, it dominates entitlements.

2. The Model

Consider a small open economy with a single sector. Interest rate, r , and average wage, \bar{w} are exogenously given and constant over time. At each period of time t , two generations overlap, L_t workers and L_{t-1} retired persons with $L_t = L_{t-1}(1 + n_t)$, where n_t is the rate of population growth between periods $t - 1$ and t . By definition, a worker born at the beginning of period t belongs to generation t . Individuals differ in two ways: the generation they belong to and their wage w , a continuous variable with mean \bar{w} , median w_m and support $[w_-, w_+]$. The cumulative distribution function and the density function are respectively denoted by $F(\cdot)$ and $f(\cdot)$. Labour supply is given and normalized at one. Finally, we assume dynamic efficiency: $r \geq n_t \geq 0, \forall t$.

The pension benefit that an individual with earning w in period t expects to receive in period $t + 1$ is denoted by $p_{t+1}^e(w)$. We assume that $p_{t+1}^e(w)$ consists of two parts: a “contributory” part which is directly related to individual earning, w , and a “noncontributory” part which is related to average earnings, \bar{w} . With a PAYG scheme, the rate of return is the population growth. All these features yield the following expression for $p_{t+1}^e(w)$:

$$p_{t+1}^e(w) = (1 + n_t)\tau_t\alpha w + p_{t+1}^o \tag{1}$$

where τ_t is the payroll tax rate in period t and α is the Bismarckian factor, that is the fraction of pension benefits that is related to contributions; we assume $0 \leq \alpha \leq 1$. When $\alpha = 0$, pension benefits are uniform and the scheme is labeled Beveridgean. A positive α means that part of the benefits are linked to individuals’ past earnings and the system is labeled Bismarckian. The higher α , the more Bismarckian is the pension system. The noncontributory part of pensions, p_{t+1}^o , is then obtained by solving the government budget constraint in period $t + 1$:

$$p_{t+1}^o = (1 + n_{t+1})\tau_{t+1}\bar{w} - (1 + n_t)\tau_t\alpha\bar{w}. \tag{2}$$

It is important to emphasize that the pension individuals working in period t expect to receive in period $t + 1$ depends on the tax rate and the rate of population growth in period t . This Bismarckian part of pensions is determined by the parameter α . The Beveridgean part of pensions is then a residual; it is equal to the difference between the revenue collected in period $t + 1$ and the contributory part of pensions paid to the retired individuals.¹

We assume throughout this paper that even though the retired individuals of generation t expect to receive a pension benefit given by (1), they do not perceive its two components the same way. The Bismarckian part results from a contractual arrangement agreed upon by

unions and management within paritarian negotiations. The Bismarckian benefits are based on wages and more generally on labor market agreements and insurance principles. The Beveridgean part on the other hand is a component of the social assistance program, which is inspired by considerations of justice and altruism. Traditionally, they do not lead to strong feelings of entitlements and have a level of legitimacy lower than that of the Bismarckian social insurance.

Let us now analyze the optimal saving decision of a working individual born in period t with earning w . He is subject to a payroll tax τ_t and expects the future tax rate to be τ_{t+1} when old. He can then allocate his disposable income between consumption c_t and saving s_t , restricted to be nonnegative.² When he retires his consumption d_{t+1} is equal to the gross return of his saving, $(1+r)s_t$, and a pension p_{t+1}^e . Formally, he solves the following program:

$$\max_{c_t, s_t, d_{t+1}} U_t = u(c_t) + \beta u(d_{t+1}) \quad (3)$$

subject to:

$$w(1 - \tau_t) = c_t + s_t, \quad (4)$$

$$s_t \geq 0 \quad (5)$$

and

$$d_{t+1} = (1+r)s_t + p_{t+1}^e(w). \quad (6)$$

In (3), $u(\cdot)$ is strictly concave and $\beta \leq 1$ is a factor of time preference. Let σ denote the elasticity of substitution between c_t and d_{t+1} . We assume throughout the paper that $\sigma < 1$, which means that there is not much substitution in consumption. We assume also that the utility function is isoelastic, that is $u(x) = x^{1-\varepsilon}/(1-\varepsilon)$. A useful property of this utility function is that the coefficient of relative risk aversion, $R_r(x) = -xu''(x)/u'(x)$ is constant and equal to ε . Moreover, it is well known that $\sigma = 1/\varepsilon$.

The first-order condition associated to an interior solution of s_t is the following:

$$-u'(c_t) + \beta u'(d_{t+1})(1+r) = 0. \quad (7)$$

Denoting $s_t^A \geq 0$ the optimal value of s_t , we can define the (indirect) utility function of an individual with income w as:

$$V_t(\tau_t, \tau_{t+1}, w) = u(w(1 - \tau_t) - s_t^A) + \beta u((1+r)s_t^A + p_{t+1}^e(w)). \quad (8)$$

3. Majority Voting in the Steady State

In Casamatta et al. (2000b), we determine the majority voting equilibrium tax rate τ^* (Condorcet winner) in the steady state, for given values of the parameters of the model, α , r and n . The steady state assumption implies that individuals cast their vote by assuming $\tau_t = \tau_{t+1}$. An individual's voting behavior is then directly determined by (8) which is a crucial ingredient of our analysis.

We prove that preferences over tax rates are single-peaked, which ensures, by the median voter theorem, the existence of a Condorcet winner. Part of the workers (the richer ones)

would like to abandon the PAYG system, which redistributes from the rich to the poor. The other part of the workers sustain a positive tax rate and, with a low intertemporal elasticity of substitution, have preferred tax rates increasing with income. It follows that a positive tax rate may be sustained by a coalition of the retired persons and the middle income workers. However, for $r > n$, τ^* is discontinuous in α . A system with a high α is poorly redistributive and it is not redistributive at all when $\alpha = 1$. Besides, the rate of return of private savings, r , is higher than the “intrinsic” rate of return of the PAYG system, n . Consequently, as α gets higher, less and less young individuals support the PAYG system and there exists a threshold value of α such that the equilibrium tax rate drops to 0 for α higher than this threshold. On the contrary, τ^* is continuous in α when $r = n$. The rates of return are identical and only the redistributive nature of the PAYG pension system makes a difference. All the individuals with $w < \bar{w}$ vote for a positive tax rate while all the individuals with $w > \bar{w}$ vote for a zero tax rate and this is so whatever the value of α .³

Comparative statics of τ^* with respect to α and n give indeterminate results. Specifically, a *marginal* decrease in population growth does not necessarily reduce the majority equilibrium contribution rate. However, we established that, under the condition that $w_- > \bar{w}/(1+r)$, the tax rate falls to 0 for a *large* decline in fertility. We also know that, when $r = n$, the equilibrium tax rate is increasing in α . This is because when $r = n$, the decisive voter is always the same, whatever the value of α . Consequently the effect a change in α is determined by its impact on the preferred tax rate of the decisive voter, which is positive.⁴

But this does not really matter. The equilibrium tax rate depends on the expectation of steady state, in particular on the expectation of constant n and α . Even though one is never sure of such constancy, voters can think that even if when they retire n or α changes, they will not receive less pensions than anticipated. In other words, they believe that they are entitled to p^e , such as defined in (1).

4. Demographic Shock

We now suppose that we suddenly move from a setting where $n_b = r$ to one with $n_a < r$ where a and b stands for after and before the shock. We adopt the convention that the shock occurs at the beginning of period 1. Generation 0 is then the generation of people who are young before the shock and old after the shock while generation 1 consists of individuals born just after the shock.

Given this decline in fertility, the new majority votes for the abandonment of the PAYG system ($\tau_a^* = 0$). If this decision were implemented, the retired persons of generation 0 would be left with no resources. We examine some mechanisms restricting the majority voting process, thereby protecting the transition generation against the “tyranny” of the majority. The mechanisms considered are the presence of entrenched interest or the existence of a guaranteed minimum pension.

4.1. *Entrenched Interests and Entitlements*

The kind of entrenched interest we have in mind is related to the concept of entitlement. Accordingly, retired persons feel entitled to a given replacement ratio depending on their

past contributions. We assume in this section that the retired individuals are able to impose the payment of those entitlements against the will of the majority and thus hold a political weight stronger than their mere number indicates.⁵ From (1), this replacement ratio is given in period $t + 1$ by:

$$\rho_{t+1} = (1 + n_t)\alpha\tau_t. \quad (9)$$

This means that a retired person in period $t + 1$ with past earnings w receive a pension *at least* equal to $\rho_{t+1}w$.

We use the notation τ_t^c for the tax rate in period t that is necessary to pay for entitlements. This is given by the following formula:

$$\tau_t^c = \frac{(1 + n_{t-1})}{(1 + n_t)}\alpha\tau_{t-1}. \quad (10)$$

The government will adopt a solution that strikes a compromise between the choice of the majority, $\tau_a^* = 0$, and that of the entrenched interest, τ_t^c . Recent work, which shows that the government has to take into account pressure from interest groups that can affect the normal course of election or threaten reelection, argues for this compromised solution. We can write this solution:

$$\tau_t = \varphi\tau_t^c,$$

where φ is the weight of retired persons in the decision process. It can be shown to be a function of the expected loss that a retired person would incur if the majority's will prevails. In what follows, we simply assume $\varphi = 1$ and thus focus on τ_t^c .

The tax rate implemented in period $t \geq 1$ is then

$$\tau_t^c = \frac{(1 + n_b)\alpha^t\tau_b^*}{1 + n_a} \quad (11)$$

and the pension received by an individual with past earnings w

$$p_t(w) = (1 + n_b)\alpha^t\tau_b^*w. \quad (12)$$

As emphasized above, the effective tax rate following the shock is the constrained tax rate τ_t^c . Every retired person receives a pension proportional to his past income. Put differently, the Beveridgean part of pensions is set to 0 and there is no intragenerational redistribution. One clearly sees that the constrained tax rate decreases, at a rate equal to $1 - \alpha$.⁶ Therefore, the PAYG system slowly disappears and the economy moves to a new steady state with no pension system. All individuals only rely on private savings to transfer resources between first and second periods.

The important point here is that when the majority would like to lower substantially the size of the PAYG system or even to abandon it, the presence of entitlements slows down this process and delays convergence to a new steady state. Consequently, those entitlements imply a better sharing of the burden of transition between generations. More specifically, the transition generation benefits from the protection of the entitlement and does no longer support the entire burden of the shock. Entrenched interests are generally viewed as an obstacle to socially desirable reform. Here, we argue that there are situations where they can be defended on ethical grounds.

4.2. Basic Pension

We now consider an alternative mechanism restricting the majority voting process, namely the possibility of a minimum pension level guaranteed to all individuals. This means that the pension cannot fall below a given level $\underline{p} > 0$ and that the constraint $p_t(w) \geq \underline{p}$ is imposed for all w and t . Therefore, the pension an individual with income w will receive in any period t is $\max \{(1 + n_{t-1})\tau_{t-1}\alpha w, \underline{p}\}$. For consistency, we assume that this constraint is satisfied in the steady state before the shock, that is $\tau_b^*(1 + n_b)(\alpha w_- + (1 - \alpha)\bar{w}) \geq \underline{p}$. The general case being rather complicated, we first develop in the following the dynamic properties of a system with $\alpha = 0$ and $\underline{p} > 0$. We obtain that the tax rates and the pension level in period $t \geq 1$ are respectively:

$$\tau_t^c = \frac{\underline{p}}{\bar{w}(1 + n_a)} \tag{13}$$

and

$$p_t(w) = \underline{p}. \tag{14}$$

The immediate consequence of such a basic pension system is that, even if $\tau_a^* = 0$, which means that a majority of the population is in favor of the abandonment of the PAYG system, this system will go on forever.⁷ Like the entitlement to the contributive part it can provide some protection for the transition generation in case of a shock, against an “expropriation” of its expected benefits by the majority. Consequently, it may be effective in inducing a better sharing of the burden of a shock between generations. Observe however that, unlike the entitlement case, in which the PAYG system slowly disappears when the new majority wants to turn it down, the PAYG system never disappears when a minimum pension is imposed.

Next, we give the formula for the constrained tax rate in the general case where $\alpha > 0$ and $\underline{p} > 0$, without specifying the full dynamics:

$$\tau_t^c = \frac{\underline{p}F(w_{o,t}) + (1 + n_{t-1})\tau_{t-1}\alpha \int_{w_{o,t}}^{w_+} wf(w)dw}{\bar{w}(1 + n_a)}, \tag{15}$$

where $w_{o,t}$ is such that $(1 + n_{t-1})\tau_{t-1}\alpha w_{o,t} = \underline{p}$. This expression specifies the tax rate that must be set in order to give every individual a pension equal to $\max \{(1 + n_{t-1})\tau_{t-1}\alpha w, \underline{p}\}$.

5. Choosing the Parameters α and \underline{p}

So far, we have assumed that the restrictions to the majority voting process were given exogenously. However, the property that both types of restrictions can be welfare-improving suggests that it might be desirable to effectively create such restriction, for instance through a constitutional choice of α and/or \underline{p} .

Observe that the way these two types of restriction can be induced is quite different. The minimum pension level would simply be set at the constitutional stage and imposed on all subsequent generations. The case of entitlements is more subtle. They are controlled only indirectly by the constitutional decision makers, through the determination of the

contributive part. In other words, it is the constitutional choice of the type of system (the degree to which it is Bismarckian) which determines the relevance of the entitlement.

5.1. Choosing α

In this section, we determine the value of α chosen at a constitutional stage under a Rawlsian criterion. This amounts to maximize the utility of the worse-off generation when the possibility of a bad shock on n is taken into account. Formally, we solve the program:

$$\max_{\alpha} \min_t \{V_t(\tau_t, \tau_{t+1}, w_-)\}, \quad (16)$$

with $V_t(\tau_t, \tau_{t+1}, w_-)$ defined in (8). In the following, we denote $V_t(\tau_t, \tau_{t+1}, w_-)$ by $V_t(w_-)$. We also assume that $n_a = 0$. In other words we assume a large shock. In this section, we consider the case where $p = 0$.

We argue now that the worse-off individuals are going to be those with the lowest wage rate and belonging to the transition generations; namely, those who are retiring or working in period 1. Generation t utility is given by:⁸

$$V_0(w_-) = u((1 - \tau_b^*)w_-) + \beta u(\alpha \tau_b^*(1 + n_b)w_-), \quad (17)$$

$$V_t(w_-) = \max_s u((1 - \tau_t^c)w_- - s) + \beta u(\alpha \tau_t^c w_- + s(1 + r)) \quad t = 1, 2, \dots \quad (18)$$

First, the lowest wage individuals are always better off in the steady state before the shock than in the transition period. Indeed, the steady state utility level before the shock, is:

$$V_{-1}(w_-) = u((1 - \tau_b^*)w_-) + \beta u(\tau_b^*(1 + n_b)(\alpha w_- + (1 - \alpha)\bar{w})). \quad (19)$$

Comparing expressions (17) and (19), one immediately sees that $V_0(w_-) < V_{-1}(w_-)$ except at $\alpha = 1$, where $V_0(w_-) = V_{-1}(w_-)$.

Second, utility increases after period 1. Indeed, one can easily verify that $\partial V_t(w_-)/\partial \tau_t^c < 0$, $\forall t \geq 1$; because the PAYG pension system offers a return of $\alpha \leq 1$ and is not redistributive anymore, individuals would be better off with no pension system, they would prefer to rely exclusively on private savings. Reminding that $\tau_t^c = (1 + n_b)\alpha^t \tau_b^*$ decreases over time, life cycle utility rises and converges to $V_{\infty}(w_-) = \max_s u(w_- - s) + \beta u(s(1 + r))$. We can then conclude that the worse-off generations are those with the lowest wage and belonging to the transition generations.

The problem for the Rawlsian constitutionalist is then to find the value of α that maximizes the minimum of either $V_0(w_-)$ or $V_1(w_-)$. We prove in Appendix 1 that $V_0(w_-)|_{\alpha=0} < V_1(w_-)|_{\alpha=0}$ and $V_0(w_-)|_{\alpha=1} > V_1(w_-)|_{\alpha=1}$. Utility levels being continuous in α , there is at least one positive value of α that equates those utility levels. If there exists several values of α with this property, the Rawlsian criterion selects the one that gives the highest level of utility.

Summing up, we have shown that a positive α may be an effective way to protect the members of the transition generation when the new majority is ready to “expropriate” them. Specifically it enhances the welfare of the poor in the transition generation even though it

makes the system less redistributive. This is because the entitlement is associated with the contributive part; a positive level of α is thus an indirect way to effectively create some entitlement.⁹

In an earlier paper by Casamatta et al. (2000a) that deals with social insurance, the choice of the Bismarckian parameter, α , is made at the constitutional level upon the expectation that the payroll tax is determined later through majority voting. One of the results of that paper is that a positive Bismarckian parameter could be desirable, even though with full control of both α and τ , the constitutional planner would choose α equal to zero. The forces that drive this result are however quite different from the ones playing here. The setting is static and corresponds to what is called here the steady state. Because the majority voting tax rate depends on the value of α chosen by the constitutional planner, this latter has an *indirect* control over the value of the tax rate. By letting α to be positive, the decisive voter will vote for a tax rate that fits better the preferences of the low wage individuals. A Rawlsian constitutionalist can then favor such a positive α .

5.2. Choosing \underline{p} and α

We now consider the situation where the constitutional body can choose both α and \underline{p} so as to maximize the welfare of the worse-off individual. This gives one additional instrument. Indeed with just α which could have been fixed at a rather high level, the intergenerational imbalance caused by the shock was taken care of, but intragenerational redistribution was made impossible.

The question raised at this point is whether we need a combination of \underline{p} and α or just one of the two and then which one. We show in Appendix 2 that *the optimal system from a Rawlsian perspective involves the choice of a zero α and a strictly positive basic pension*. The intuition is simple: for a given benefit level, the lowest wage earner pays less if it is provided through \underline{p} than through individualized contribution. With $\alpha = 0$, redistribution within each generation becomes possible.¹⁰

It is however very likely that with an objective function less extreme than the Rawlsian criterion and with distortionary taxation there is room for a positive α .

The optimal value of \underline{p} can be obtained by equating $V_0(w_-)$ and $V_1(w_-)$. We show in Appendix 3 that such a solution exists and is unique. It is the value of \underline{p} that solves the following equation:

$$\begin{aligned}
 &u((1 - \tau_b^*)w_-) + \beta u(\underline{p}) \\
 &= \max_s u\left(\left(1 - \frac{\underline{p}}{\bar{w}}\right)w_- - s\right) + \beta u(\underline{p} + s(1 + r)).
 \end{aligned}$$

If it implies no saving, the optimal \underline{p} is equal to $\tau_b^*(0)\bar{w}$. Moreover, whatever the optimal value of \underline{p} , the utility $V_t(w_-)$ for $t = 0, 1, \dots$ will be constant and given by:

$$V_t(w_-) = \max_s u\left(\left(1 - \frac{\underline{p}}{\bar{w}}\right)w_- - s\right) + \beta u(\underline{p} + s(1 + r)) \quad t = 0, 1, \dots \tag{20}$$

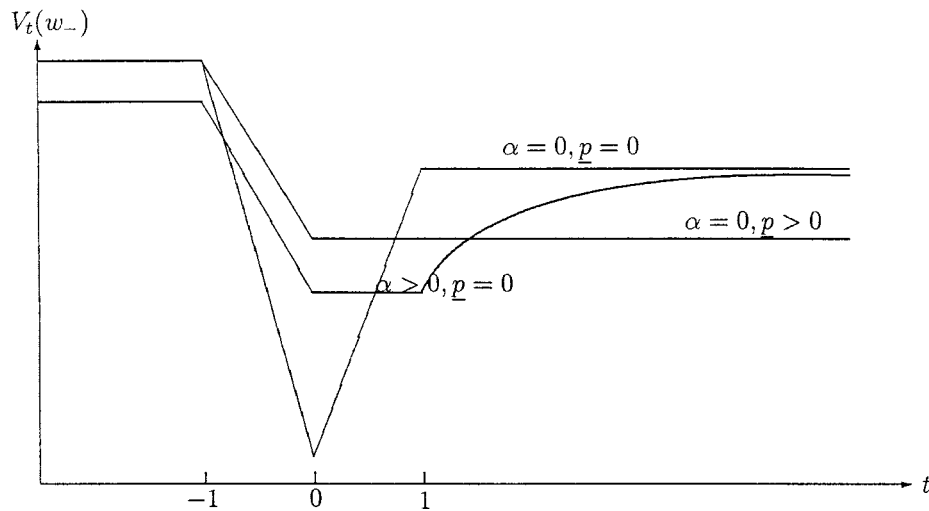


Figure 1. Life cycle utility of the lowest income individuals under alternative PAYG systems.

These results and, more generally, the welfare impact of α and \underline{p} are illustrated on Figure 1. This figure shows lifetime utilities of the poorest individuals in successive generations.

First, $\alpha = 0$, $\underline{p} = 0$ corresponds to the case where there is no contributive part (and hence no entitlement) and no minimum pension. In this case, the poor of the transition generation bear all the burden of the shock and have a low utility level. Transition to the new steady state (with no PAYG pension) takes only one period.

The second curve corresponds to $\underline{p} = 0$ (no minimum pension) while $\alpha > 0$ is optimally chosen from the Rawlsian perspective. Consequently, there is a contributive part to which the transition generation is entitled. Observe that the optimal α equalizes the utilities of the worst-off individuals of generations 0 and 1. Now the transition generation is hit less severely, but transition to the new steady state is delayed. Subsequent generations bear an efficiency cost which asymptotically tends to 0.

Finally, we also represent the solution when the minimum pension is available. Then we have $\alpha = 0$ and $\underline{p} > 0$. The Rawlsian social welfare function implies that the minimum pension is chosen to equalize the utility of the poorest individuals in generations 0 and 1. In this situation, the worst-off individual in the transition generation is hit less severely than in the entitlement case, but now the welfare cost on future generations will not vanish in the long run.¹¹

6. Conclusion

We have investigated in this paper the issue of the political support for a PAYG system in the case of a negative demographic shock. The evolution of the system depends on the assumptions made concerning the type of system considered and the decision making process. When the new majority chooses the payroll tax rate following the shock, without

any constraint, the economy switches immediately to a new steady state. In particular, when the new majority wants to abandon the system, it is dismantled and individuals rely exclusively on private savings to smooth their consumption across time. We have identified two mechanisms allowing to constrain the choice of the majority following the shock, thereby protecting the retired persons of the transition generations. The first one is linked to the existence of entrenched interests. Those entrenched interests are generally viewed as an obstacle to socially desirable reforms. However, they may be a good way to cope with the unfairness of the political process. The second mechanism is simply the existence of a minimum level of benefit that can be determined by a Rawlsian constitutional planner.

This planner can not only determine the level of a minimum pension but also the degree in which social security benefits are linked to earnings, what we call the Bismarckian parameter. The importance of entrenched interests is indeed linked to that parameter. If the planner can determine both the minimum pension and the Bismarckian parameter, we show that the minimum pension is a better instrument from a Rawlsian viewpoint. If only the Bismarckian parameter is available, then it can be fixed at a rather high level so preventing the social security system to effect intragenerational redistribution.

Here we assume that both α and \underline{p} are chosen at the constitutional level and henceforth are not subject to subsequent revision. Admittedly, the case for α is stronger than that of \underline{p} . One can argue that for the same reasons as the Beveridgean part of social security benefits does not lead to robust entitlements, one can expect that \underline{p} be subject to political attack and thus to progressive erosion.

This paper leaves a number of unanswered questions. First, we would like in future research to endogenize the emergence of entrenched interests. Second, the assumption of commitment, that is the assumption that workers vote with the belief that the economy is in steady-state and that the tax rate chosen today will apply to them when retired, is somehow questionable. Further, individuals are rather crude in making their decisions concerning the future. In particular, the assumption that the workers believe to be in a new steady state following the shock needs to be modified. Finally, we have to consider an objective less restrictive than the Rawlsian criterion; we know that it has serious limits for problems involving intergenerational redistribution.

Appendix

A. Appendix 1

We verify in this appendix that $V_0(w_-)|_{\alpha=0} < V_1(w_-)|_{\alpha=0}$ and $V_0(w_-)|_{\alpha=1} > V_1(w_-)|_{\alpha=1}$.

First,

$$V_0(w_-)|_{\alpha=0} = u((1 - \tau_b^*)w_-) + \beta u(0) \quad (21)$$

and

$$V_1(w_-)|_{\alpha=0} = \max_s u(w_- - s) + \beta u(s(1 + r)). \quad (22)$$

Clearly, the expression (22) is higher than (21). The reason is that generation 1 individuals could achieve a higher level of utility than generation 0 by setting $s = 0$.

Second,

$$V_0(w_-)|_{\alpha=1} = u((1 - \tau_b^*)w_-) + \beta u(\tau_b^*(1 + n_b)w_-) \quad (23)$$

and

$$V_1(w_-)|_{\alpha=1} = \max_s u((1 - \tau_b^*(1 + n_b))w_- - s) + \beta u(\tau_b^*(1 + n_b)w_- + s(1 + r)). \quad (24)$$

Recalling that $n_b = r$ and that $\partial \tau^A / \partial w|_{\alpha=1} = 0$,¹² we have:

$V_0(w_-)|_{\alpha=1} = \max_s u(w_- - s) + \beta u(s(1 + r))$ which is necessarily higher than the expression in (24).

B. Appendix 2

Consider a situation with $\alpha = \tilde{\alpha} > 0$ and $\underline{p} = \tilde{p} \geq 0$. We show that the utility of generations 0 and 1 can be raised by setting $\alpha = 0$ and an appropriate level of \underline{p} .

First, we write:

$$\tilde{V}_0(w_-) = u((1 - \tau_b^*(\tilde{\alpha}))w_-) + \beta u(\max\{(1 + n_b)\tau_b^*(\tilde{\alpha})\tilde{\alpha}w_-, \tilde{p}\}) \quad (25)$$

and

$$\tilde{V}_1(w_-) = \max_s u((1 - \tilde{\tau}_1^c)w_- - s) + \beta u(\max\{\tilde{\tau}_1^c \tilde{\alpha}w_-, \tilde{p}\} + s(1 + r)). \quad (26)$$

Two cases must then be considered.

Case 1: $\tilde{\alpha}\tau_b^*(\tilde{\alpha})(1 + n_b)w_- \geq \tilde{p}$

In this case, one has $\tilde{\tau}_1^c = (1 + n_b)\tau_b^*(\tilde{\alpha})\tilde{\alpha}$. Let us replace the old system by a new one with $\alpha = 0$ and $\underline{p} = \tilde{\alpha}\tau_b^*(\tilde{\alpha})(1 + n_b)w_-$. Because $n_b = r$, we know that $\partial \tau_b^*(\alpha) / \partial \alpha > 0$. Therefore, $\tau_b^*(\tilde{\alpha}) > \tau_b^*(0)$ which implies that

$$\tilde{c}_0 = (1 - \tau_b^*(\tilde{\alpha}))w_- < c_0 = (1 - \tau_b^*(0))w_-. \quad (27)$$

Moreover,

$$\tilde{d}_1 = \tilde{\alpha}\tau_b^*(\tilde{\alpha})(1 + n_b)w_- = d_1. \quad (28)$$

Generation 0 is thus strictly better off with the new system. Let's examine now generation 1 welfare. With the new system, $\tau_1^c = \underline{p}/\tilde{w} = \tilde{\alpha}\tau_b^*(\tilde{\alpha})(1 + n_b)w_-/\tilde{w} < \tilde{\tau}_1^c$. Hence, omitting the terms relative to savings,

$$\tilde{c}_1 = (1 - \tilde{\tau}_1^c)w_- < c_1 = (1 - \tau_1^c)w_- \quad (29)$$

and

$$\tilde{d}_2 = \max\{\tilde{\alpha}^2\tau_b^*(\tilde{\alpha})(1 + n_b)w_-, \tilde{p}\} \leq d_2 = \tilde{\alpha}\tau_b^*(\tilde{\alpha})(1 + n_b)w_-. \quad (30)$$

We can conclude that generation 1 is also strictly better off.

Case 2: $\tilde{\alpha}\tau_b^*(\tilde{\alpha})(1+n_b)w_- < \tilde{p}$

Now,

$$\tilde{\tau}_1^c = \frac{\tilde{p}F(w_{o,1}) + (1+n_b)\tau_b^*(\tilde{\alpha})\tilde{\alpha} \int_{w_{o,1}}^{w_+} wf(w)dw}{\bar{w}}. \quad (31)$$

Let us replace $\tilde{\alpha}$ and \tilde{p} by $\alpha = 0$ and $\underline{p} = \tilde{p}$. Then we have $\tau_1^c = \underline{p}/\bar{w} \leq \tilde{\tau}_1^c$ and the following inequalities hold:

$$\tilde{c}_0 = (1 - \tau_b^*(\tilde{\alpha}))w_- < c_0 = (1 - \tau_b^*(0))w_-, \quad (32)$$

$$\tilde{d}_1 = \tilde{p} = d_1, \quad (33)$$

$$\tilde{c}_1 = (1 - \tilde{\tau}_1^c)w_- \leq c_1 = (1 - \tau_1^c)w_- \quad (34)$$

and

$$\tilde{d}_2 = \tilde{p} = d_2. \quad (35)$$

C. Appendix 3

With a system $\alpha = 0$ and $\underline{p} > 0$, we have:

$$V_0(w_-) = u((1 - \tau_b^*)w_-) + \beta u(\underline{p}) \quad (36)$$

and

$$V_1(w_-) = \max_s u\left(\left(1 - \frac{\underline{p}}{w}\right)w_- - s\right) + \beta u(\underline{p} + s(1+r)). \quad (37)$$

First, it is obvious that $\partial V_0(w_-)/\partial \underline{p} > 0$. Second, we argue that $V_1(w_-)$ is strictly concave in \underline{p} . This is easily verified when writing the second order derivative. In order to show that there is a unique optimal \underline{p} , we must now show that $V_0(w_-)|_{\underline{p}=0} < V_1(w_-)|_{\underline{p}=0}$ and that $V_0(w_-)|_{\underline{p}=\underline{p}^{\max}} > V_1(w_-)|_{\underline{p}=\underline{p}^{\max}}$, where $\underline{p}^{\max} = \tau_b^*\bar{w}(1+n_b)$. The first inequality is obvious. Consider the second inequality. We can write:

$$V_0(w_-)|_{\underline{p}=\underline{p}^{\max}} = u((1 - \tau_b^*)w_-) + \beta u(\tau_b^*\bar{w}(1+n_b)) \quad (38)$$

and

$$V_1(w_-)|_{\underline{p}=\underline{p}^{\max}} = \max_s u((1 - \tau_b^*(1+n_b))w_- - s) + \beta u(\tau_b^*\bar{w}(1+n_b) + s(1+r)). \quad (39)$$

Recalling that the lowest wage individuals do not want to save in the steady state before the shock, we obtain that $V_0(w_-)|_{\underline{p}=\underline{p}^{\max}} > V_1(w_-)|_{\underline{p}=\underline{p}^{\max}}$.

Finally, the value of \underline{p} chosen by the Rawlsian authority is such that $V_0(w_-)$ and $V_1(w_-)$ are equalized. Indeed, those two curves necessarily intersect on the decreasing portion of $V_1(w_-)$. This follows from the observation that $V_1(w_-)|_{\underline{p}=\underline{p}^*} > V_0(w_-)|_{\underline{p}=\underline{p}^*}$, where $\underline{p}^* = \arg \max_{\underline{p}} V_1(w_-)$.

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Notes

1. In the steady state, where $n_t = n$ and $\tau_t = \tau$, $\forall t$, the benefit formula becomes:

$$p^e(w) = (1 + n)\tau(\alpha w + (1 - \alpha)\bar{w}).$$

2. Allowing negative savings would generate extreme solutions. Young people in favor of the PAYG pension system would vote for a tax rate of 1 and would rely entirely on borrowing to finance their present consumption.
3. At $\alpha = 1$, every young worker is indifferent and we make the assumption that he supports the public pension system. This ensures the continuity of τ^* at this point.
4. This follows from the assumption that the intertemporal elasticity of substitution is small. Consider an individual, such as the median voter, with a wage below the average wage. When α increases, the pension system becomes less redistributive making second period consumption more expensive relative to first period consumption. The substitution effect then calls for an increase in first period consumption and thus for a lower tax rate. However, with a small elasticity of substitution the income effect, which goes in the opposite direction, dominates so that the considered individual chooses a higher tax rate.
5. See on this Disney (1996). We do not explain in this paper how the retired persons impose their choice to the majority, we just assume it. Some explanations rely on the lobbying activity of the retired individuals (see Grossman and Helpman (1998) or Mulligan and Sala-i-Martin (1999)) or on the multi-dimensional nature of the political process (see Casamatta, 2000).
6. Except of course when $\alpha = 1$ in which case it is constant.
7. Alternatively, the social security system disappears, but is replaced by a particular kind of negative income tax scheme under which the flat benefit is restricted to the elderly.
8. One can show that the poorest individuals do not want to complement the public pension with private saving in the steady state before the shock. This implies that there is no term related to savings in the first following equation.
9. The level of α we determine here maximizes the Rawlsian welfare in case of a shock *within the class of considered decision rules*. This does not imply that the political process in itself is optimal. Nor does it imply that it is necessarily desirable to impose a constitutional constraint on α . As pointed out by the referee, society may indeed be able to find a welfare enhancing arrangement through a suitable renegotiation of the benefit formula after the shock. This raises the challenging problem of the design of political institutions and of the role of wide-based negotiations as a complement to voting procedures in a democracy. Studying these issues is an ambitious endeavor which goes well beyond the scope of this paper.
10. Note that, when $n = r$, $\alpha = 0$ is also the solution that would choose a Rawlsian constitutionalist in the steady state. Indeed, denoting $v - (\alpha) = V(\tau^*, \tau^*, w_-)$ and recalling that $d\tau^*/d\alpha > 0$ when $n = r$, we have:

$$\frac{dv_-}{d\alpha} = \underbrace{\frac{\partial V}{\partial \tau}}_{<0} \frac{d\tau^*}{d\alpha} + \underbrace{\frac{\partial V}{\partial \alpha}}_{<0} < 0.$$

11. A sufficient condition for this result is that $w_- > \bar{w}/(1 + r)$. When this condition is satisfied, the return from private saving, $1 + r$, dominates the return from PAYG pensions, \bar{w}/w_- , from the point of view of the lowest wage individuals. Note that in the opposite case, $w_- \leq \bar{w}/(1 + r)$, it is still possible that the lowest wage individuals prefer private savings. This comes from the fact that the level of PAYG pensions is not set at their preferred level.
12. This is established in Casamatta et al. (2000b). Note that the argument developed here holds in the more general case where $\partial \tau^A / \partial w|_{\alpha=1} \geq 0$.

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