J Popul Econ 19:345–365 (2006)
DOI 10.1007/s00148-006-0063-1

ORIGINAL PAPER

**Panu Poutvaara**

# On the political economy of social security and public education

Received: 29 October 2004 / Accepted: 23 November 2005 / Published online: 1 April 2006


**Abstract** This paper analyzes simultaneous voting on the wage tax rate and investment in public education with three overlapping generations and ability differences inside each cohort. Wage tax revenue finances public education and social security benefits. The presence of ability differences introduces a time-consistency problem with repeated voting. This can be solved by trigger strategies, which do not punish upward deviations in the wage tax rate. If there are multiple equilibria, then higher tax rates are associated with more education. Surprisingly, the median voter may be a young citizen, even when cohorts are of the same size.

**Keywords** Social security · Public education · Voting

**JEL Classification** H52 · H55 · D72

## 1 Introduction

Important and often overlooked aspects in reforming social security are political linkages between different public expenditures. Education and social security are

*Responsible editor*: Gil S. Epstein

The first draft of this paper appeared as part of my doctoral dissertation at the University of Helsinki. An earlier version was written while I was a Marie Curie Fellow at CES at the University of Munich, whose hospitality I gratefully acknowledge. The paper was revised while I was employed full-time at the Centre for Economic and Business Research at the Copenhagen Business School. Earlier versions of this paper were presented at the ENTER Jamboree in Mannheim, February 14–17, 2001, the European Public Choice Society Annual Meeting in Paris, April 18–21, 2001, the 57th International Institute of Public Finance Congress in Linz, August 27–30, 2001, the CEPR/EPRU Workshop "Dynamic Aspects of Public Expenditure" in Copenhagen, September 28–30, 2001, and at the EEA Annual Meeting in Venice, August 22–24, 2002.

P. Poutvaara (✉)
Department of Economics, University of Helsinki, Arkadiankatu 7, P.O. Box 17, 00014 Helsinki, Finland
Fax: +358-9-19128736, E-mail: panu.poutvaara@helsinki.fi

amongst the most important public expenditures in most developed countries. The education system to a large extent determines future human capital and is, therefore, crucial to productive capacity. Social security benefits are a major form of redistribution in most countries, taking in an average of 8.8% of the GDP in the member states of the European Union in 2001 (OECD 2004a). At the same time, public expenditures on higher education took in an average of 1.4% of GDP (OECD 2004b). These systems interact in individual and political decision-making, as redistribution through social security gives lower-productivity individuals a claim on the future productivity of high-productivity individuals. Boldrin and Montes (2002) formalize public education and pay-as-you-go social security as two parts of an intergenerational contract. They assume identical voters inside each cohort and show that public education can be maintained by linking it into a sufficiently large social security system. Rangel (2003) characterizes an implicit intergenerational contract on the provision of social security and goods benefiting younger generations, such as education and environment, also focusing on the case of identical individuals. Boldrin and Montes (2005) take a normative perspective and show that when credit markets to finance human capital are missing, using lump-sum taxes to finance public education and pensions can restore efficiency.

In the presence of productivity differences, an implicit intergenerational contract on financing higher education may suffer from time-consistency problem. The time-consistency problem refers to a situation in which some voters would strategically vote for an alternative that they would not vote for with commitment, with an aim to activate trigger strategies. Young high-productivity citizens may be prime beneficiaries from an intergenerational contract linking public education and social security benefits, but they would like to see such a contract unravel after completing their education and starting to work and pay taxes. Nonetheless, public education and social security have emerged and persisted in the political process in all member states of the European Union and, widely, also elsewhere. In Austria, Denmark, Finland, Greece, Norway, Poland, and Switzerland, 100% of tertiary education is publicly financed (OECD 2004c).[1]

This paper suggests a median voter model that explains how social security and public education can be maintained with repeated voting also in the presence of productivity differences. It also finds that there can be multiple equilibria. If there are multiple equilibria, then a higher provision of public education is always associated with a higher wage tax rate. This suggests that even if redistribution through a social security system causes dead-weight loss, it might still improve efficiency by resulting in a more efficient provision of higher education.

The model economy has three overlapping generations of heterogeneous citizens. The young either study or work. The middle-aged work either as educated or uneducated labor. Workers divide their time between the official labor market and home production, the latter generating endogenously a deadweight loss arising from taxation. The elderly are retirees. The wage tax revenue is used to finance both public education for the young and social security benefits for the retirees. The

[1] The share is at least 80% also in Belgium, the Czech Republic, France, Germany, Ireland, Italy, Portugal, the Slovak Republic, and Sweden. In Hungary, the Netherlands, Spain, and the United Kingdom, the share of public financing is 70 to 80%.

analysis is restricted to a linear taxation.[2] Benefits are uniform, a simplification which captures the redistributive effects of social security.[3] The citizens vote simultaneously on the wage tax rate and on the provision of public education. Voting on the wage tax rate and on the provision of public education captures competing uses of tax revenue. For instance, the elderly may benefit also from public health care, financed out of the same tax revenue as public education. With any given level of wage taxation, the elderly would then prefer not to spend resources on education, but use resources on expenditures benefiting themselves. Following Rangel (2003) and Boldrin and Montes (2002, 2005), this paper analyzes the case in which all education is financed publicly.[4]

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model. Section 4 provides a complete characterization of the consumer behavior for given levels of policy parameters. Section 5 is divided into two parts. The first part analyzes the economy when voting takes place with commitment to maintain the outcome during the lifetime of all voters. The second part generalizes all the results to an economy with repeated voting and reports what type of implicit intergenerational contract solves the time-consistency problem when voting is repeated. Section 6 concludes.

## 2 Related literature

The interaction between social security and public education in the political process has received surprisingly little attention in economics literature. Pogue and Sgontz (1977) argued that the pay-as-you-go social security system provides a more powerful incentive for the current working age generations to invest in the human capital of younger generations compared to a fully-funded social security system. By giving the current generation a stake in future production, it encourages the elderly to support such investments. Becker and Murphy (1988) suggest interpreting social security as intergenerational trade: children repay the public education they have received in old-age benefits. However, they do not study whether such an arrangement would be sustainable in the political process, which is the focus of this paper. Konrad (1995) presents an overlapping generations model in which the older generation controls the political process. He argues that the elderly have an incentive to provide education and public infrastructure to increase social security tax revenue used to finance their benefits.

Rangel (2003) and Boldrin and Montes (2002) formalize public education and pay-as-you-go social security as two parts of an intergenerational contract, and Boldrin and Montes (2005) develop the same idea from a normative perspective. In their models, all citizens inside each cohort are identical. To the best of my

[2] Voting equilibrium may not be established by general non-linear taxation because of the possibility of Condorcet cycles.

[3] Miles and Timmermann (1999) report that the gross replacement rate was more than 10 percentage points higher for low-income workers in Belgium, Denmark, France, Greece, Ireland, Luxembourg, the Netherlands, Spain, Sweden, and the United Kingdom in 1997.

[4] Boldrin and Montes (2002, 2005) also solve for a private investment equilibrium but only in the case where there are no borrowing constraints, and no taxation. Also in this paper, private investment in education would be efficient if there are no borrowing constraints and there is no taxation.

knowledge, there are no studies analyzing public education and social security arising from majority voting in the presence of ex ante visible productivity differences inside each cohort.

When policy space is multidimensional and voters may have preferences that are not single-peaked, Condorcet winners need not exist. However, the political process always imposes some restrictions on decision-making. Shepsle (1979) shows that including the voting rules can generate a structure-induced equilibrium even when unrestricted voting would not result in a Condorcet winner. A structure-induced equilibrium is a point in the policy space, which is invulnerable in the sense that with given institutional rules, there is no sufficient majority to change it. In this paper, the institutional structure imposed on political decision-making process is that citizens vote simultaneously on two issues. For example, there could be a Swiss-style referendum in which each citizen votes on each issue. Building on earlier contributions by Shepsle (1979), Cooley and Soares (1999), Boldrin and Rustichini (2000), and Conde Ruiz and Galasso (2003), this paper identifies conditions under which public education and social security can be maintained, with repeated voting, by an implicit intergenerational contract relying on subgame perfect voting strategies without commitment. The solution concept with repeated voting is stationary subgame perfect structure-induced equilibrium, developed by Conde Ruiz and Galasso (2003). This notion combines structure-induced equilibrium in a voting system with commitment to an implicit intergenerational contract.[5]

Cooley and Soares (1999) assume a trigger strategy, specifying that the voting outcome converges to a complete abolition of the social security system in the case of the result in one period differing from the subgame perfect equilibrium with a once-and-for-all voting. In their example, voters make a binary choice on whether to maintain social security. In the framework of this paper with continuous choice, a threat strategy specifying that any deviation is punished by the abolition of social security would not always maintain the subgame perfect equilibrium, resulting from voting with commitment, when there is instead repeated voting. The reason for this is that the high-productivity young citizens might find it optimal to deviate. When voting with commitment, they would prefer a lower wage tax rate than the median voter. By voting for a higher rate than the median voter, they would change the identity of the median voter. In this way, they would cause an upward deviation, triggering an elimination of wage taxation and public education.

In a similar way, a problem of deliberate deviations in the first period of life to undermine the equilibrium does not arise in Kotlikoff et al. (1988), Boldrin and Rustichini (2000), and Conde Ruiz and Galasso (2003) as they model only two overlapping generations. When the young generation is going to be retired in the following period, even young citizens losing from social security over their whole lifetime have no incentive to abandon it in the following period. Furthermore, this issue would not arise in the absence of heterogeneity with three overlapping generations, as the median voter would then always be middle-aged. Thus, the time-consistency problem is not present in the contributions by Rangel (2003) and

[5] A related concept developed to study repeated majority voting is the notion of a Dynamic Condorcet Winner, developed by Bernheim and Nataraj (2004). They assume infinitely lived agents. Using the same framework, Nataraj (2004) considers dividing a fixed surplus with overlapping generations. This paper is more general than Nataraj (2004) in endogenizing the surplus arising from public education and in allowing for ability differences, while Nataraj (2004) is more general in allowing for arbitrary divisions of the fixed pie between all cohorts.

Boldrin and Montes (2002), as they assume that all voters are identical.[6] In this way, repeated voting with three overlapping generations and heterogeneity poses qualitatively more complex challenges to reputational mechanisms than are present with two overlapping generations, with or without heterogeneity, or three overlapping generations without heterogeneity. This paper contributes to former literature in this field by suggesting a trigger strategy solving these problems.

## 3 The model

### 3.1 The economy

The focus of the analysis is a small open economy, facing a time-invariant world interest rate $r$. There is no migration. Production in period $t$ depends on the effective supply of labor services, denoted by $L_t$, and of physical capital $K_t$. Note that as capital is perfectly mobile, the stock of physical capital does not depend on domestic saving. Total production in the economy is given by Cobb–Douglas production function $Y_t = AL_t^{\alpha}K_t^{1-\alpha}$. Without loss of generality, capital does not depreciate. All markets are competitive, and, therefore, profit maximization implies that

$$w_t = \alpha A L_t^{\alpha-1} K_t^{1-\alpha}, r = (1-\alpha) A L_t^{\alpha} K_t^{-\alpha}.$$

The expression for $r$ follows from international mobility of capital. Solving for $K_t$ and inserting it into the expression for $w_t$ yields

$$w_t = \alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}.$$

Even though the production technology combines the effective labor supply and physical capital, the wage rate for a unit of effective labor does not depend on the total labor supply. This is due to the international mobility of physical capital. This time-invariant wage rate is from now on normalized to unity.

### 3.2 The citizens

Individuals differ in two dimensions: age and productivity. There are three overlapping generations in each period: the young, the middle-aged, and the elderly. There are two distinct career paths. Those who become educated study as young and work as middle-aged. Those who remain uneducated work both as young and as middle-aged. Both the educated and the uneducated are retired in the third period of their life. Education in this model corresponds to higher education, and it is provided publicly. Like Boldrin and Montes (2005), this paper assumes that consumption as young does not enter utility, and that the young cannot borrow to finance their investment in human capital. The government has access to entrance examinations,

[6] With identical voters inside each cohort and three overlapping generations of equal size, no single voter or a group of voters smaller than a half of a cohort could cause a deviation. With a continuum of types, even a single individual could cause a deviation.

which allow it to screen applicants to higher education. While not used in all countries, entrance examinations or results from baccalaureate or other tests are commonly used to select those who are admitted. The length of each period is one.

During their working life, individuals divide their time between work in the official labor market and untaxed home production.[7] We denote the amount of time that worker $i$ devotes to home production by $x_i$, implying that time spent in the official labor market is $1 - x_i$. The marginal productivity is constant in the official labor market, but the marginal productivity in home production is diminishing. The marginal productivity in home production is equal to the marginal productivity in the official labor market, multiplied by $1 - \frac{x_i}{2}$. As the length of both working periods is one, this formulation implies that the marginal productivity in the home production equals the marginal productivity in the official labor market when no time is spent in home production, then steadily declining until it reaches one-half of the productivity in the official labor market in case all time is spent in home production.

Citizens differ in their stock of human capital. The level of human capital in case of receiving higher education is denoted by variable $a$, labeled ability for simplicity. A citizen who remains uneducated has $u_1$ units of human capital as young and $u_2$ units as middle-aged, independently of ability. The wage tax rate in period $t$ is denoted by $\tau_t$ and the lump-sum social security benefits are given by $b_t$. Similar to Boldrin and Montes (2005), preferences are given by a well-behaved utility function

$$U = u\left(c^m_{i,t+1}\right) + \delta u\left(c^o_{i,t+2}\right), \tag{1}$$

where $c^m_{i,t+1}$ is consumption as middle-aged and $c^o_{i,t+2}$ is consumption as old for an individual $i$ who is young in period $t$.[8] To simplify subsequent analysis, it is assumed that the individual discounting corresponds to discounting in the financial market, so that $\delta = \frac{1}{1+r}$. The time individual $i$ who is young in period $t$ spends in home production as young (middle-aged) is denoted by $x^y_{i,t}$ ($x^m_{i,t+1}$). The individual lifetime budget constraint is for an individual $i$ who is young in period $t$ and remains uneducated

$$s^y_{i,t} = (1-\tau_t)(1-x^y_{i,t})u_1 + x^y_{i,t}\left(1-\frac{x^y_{i,t}}{2}\right)u_1, \tag{2}$$

$$c^m_{i,t+1} = (1-\tau_{t+1})(1-x^m_{i,t+1})u_2 + x^m_{i,t+1}\left(1-\frac{x^m_{i,t+1}}{2}\right)u_2 + (1+r)s^y_{i,t} - s^m_{i,t+1} \tag{3}$$

$$c^o_{i,t+2} = b_{t+2} + (1+r)s^m_{i,t+1}. \tag{4}$$

[7] Untaxed home production may also take form of working in ordinary production and evading taxes. Here the assumption corresponding to the declining marginal productivity in home production would be that an increasing fraction of time spent in tax-evading work goes to avoiding detection.

[8] Allowing consumption in the first period would not change the results, as shown in the previous version of this paper, which is available as IZA DP 1408.

Equation 2 reports that the saving in the first period consists of after-tax income from work in the official labor market and the income from home production, which is the product of time spent in home production, $x^y_{i,t}$, and marginal productivity in home production, $(1-\frac{x^y_{i,t}}{2})u_1$. Equation 3 reports consumption in the second period. The first two terms on the right-hand side are after-tax income from work in the official labor market and income from home production as middle-aged, the third term is the value of savings from the previous period, and the last-term saving made as middle-aged. The first term on the right-hand side of Eq. 4 is social security benefit, and the second term the value of private assets. If the citizen with ability $a_i$ becomes educated, the lifetime budget constraint becomes

$$c^m_{i,t+1} = (1-\tau_{t+1})(1-x^m_{i,t+1})a_i + x^m_{i,t+1}(1-\frac{x^m_{i,t+1}}{2})a_i - s^m_{i,t+1} \tag{5}$$

$$c^o_{i,t+2} = b_{t+2} + (1+r)s^m_{i,t+1}. \tag{6}$$

The budget constraint of those who become educated is simpler, as they do not work and, thus, do not earn income, in the first period. In each period, there are three overlapping generations, each of unit mass. Citizens are heterogeneous in their ability. The individual-specific ability variable follows in each period the same distribution in an interval, $a \in [0, \overline{a})$. [9] The density function of $a$ is $f(a)$ and its cumulative distribution function is $F(a)$.

### 3.3 The government's budget constraint

The government's tax revenue in any given period depends on the educational status of the middle-aged cohort, on how many of the young are going to work, and on individual labor supply decisions. Denote the ability threshold above which citizens became educated in period $t-1$ by $\widetilde{a}_{t-1}$, so that the current ability threshold is correspondingly $\widetilde{a}_t$. [10] The government's wage tax revenue is given by

$$\begin{aligned} R_t(\widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) = \quad & \tau_t(1-x^y_{u,t})F(\widetilde{a}_t)u_1 \\ & +\tau_t(1-x^m_{u,t})F(\widetilde{a}_{t-1})u_2 \\ & +\tau_t \int_{a=\widetilde{a}_{t-1}}^{\overline{a}} (1-x^m_{e,t}(a))af(a)da. \end{aligned} \tag{7}$$

[9] Assuming that there are no citizens with ability $\overline{a}$ simplifies the notation in Section 5. Voting for an ability threshold $\overline{a}$ is there equivalent to voting for a zero provision of public education.

[10] This ability threshold is either the politically decided minimum ability required to enter the higher education or the minimum ability with which an individual would like to enter higher education, whichever is higher. Section 5 shows that in all equilibria, the politically decided minimum ability required to enter the higher education is higher than the minimum ability with which an individual would like to enter higher education.

The first term gives the tax revenue received from the young who remain uneducated, the second term gives the tax revenue from the middle-aged uneducated and the third term gives the tax revenue from the middle-aged educated. The first two terms take into account that as all citizens who remain uneducated are identical, they choose the same time allocation between work in the official labor market and home production. The third term allows the labor supply of the middle-aged educated to depend on their productivity.

In each period, the government uses its tax revenue for two purposes: financing public education and financing benefits for the elderly. The marginal cost of providing public education is constant $q$. Therefore, the cost of providing public education to the young above ability threshold $\widetilde{a}_t$ is in period $t$

$$(1 - F(\widetilde{a}_t))q.$$

For simplicity, it is assumed that $q \geq ru_1$. The results can easily be generalized to $q < ru_1$ as outlined in footnote 13. Social security benefits can be solved as a function of $\tau_t$, $\widetilde{a}_{t-1}$, and $\widetilde{a}_t$:

$$b_t(\widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) = R_t(\widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) - (1 - F(\widetilde{a}_t))q. \tag{8}$$

Financing education from the tax revenue used to finance social security benefits instead of having separate tax rates for the two captures the idea of competing uses for the tax revenue. It is not required that $b_t > 0$. If $\tau_t = 0$ and education is publicly financed, $b_t$ would be negative by the government's budget constraint. However, it turns out that in equilibrium, $b_t$ is never negative. The first (second) argument of $b_t(\widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t)$ refers to the ability threshold in period $t-1$ ($t$).

## 4 Consumer behavior

Citizens have a double role in the economy. As consumers, they decide on their labor supply and savings behavior, taking as given wage tax rate and expected social security benefits. As voters, they vote on wage tax rates and public provision of education, taking into account how the political equilibrium affects their utility. In this section, we solve for the choices of individual consumers, then proceeding into voting behavior in the next section.

By Eqs. 1, 2, 3 and 5 workers choose their time allocation between work in the official labor market and home production to maximize the sum of their after-tax labor income. The young uneducated and the middle-aged uneducated and the middle-aged educated all choose the same time allocation, given by

$$x^y_{u,t} = x^m_{u,t} = x^m_{e,t}(a) = \tau_t. \tag{9}$$

By Eq. 9, the time allocated to home production is increasing in the wage tax rate. Inserting this into $R_t(\widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t)$ we find that with given education choices, the government's tax revenue first increases and then decreases in the wage tax rate, reaching maximum with $\tau_t = \frac{1}{2}$.

Both the educated and the uneducated choose their saving as middle-aged to maximize their lifetime utility. With given expectation for the benefit in the following period, $b_{t+2}$, the first-order condition reads for both the educated and the uneducated who are young in period $t$ as

$$-u'(c^m_{i,t+1}) + \delta(1+r)u'(c^o_{i,t+2}) = 0.$$

By $\delta(1+r) = 1$, the saving as middle-aged is for those who remain uneducated

$$s^m_{u,t+1} = \frac{(1-\tau_t + \frac{\tau_t^2}{2})(1+r)u_1 + (1-\tau_{t+1} + \frac{\tau_{t+1}^2}{2})u_2 - b_{t+2}}{2+r} \tag{10}$$

and for those who become educated

$$s^m_{i,t+1} = \frac{(1-\tau_{t+1} + \frac{\tau_{t+1}^2}{2})a - b_{t+2}}{2+r}. \tag{11}$$

Saving is used to equalize consumption across the two periods. As long as the lifetime income of the educated is higher than the lifetime income of the uneducated, the educated save more as middle-aged. Also, the educated save more the higher their productivity.

The decision on whether to accept public education is based on whether this results in a higher net present value of lifetime income. Taking into account Eq. 9, a young citizen $i$ would like to accept higher education, if offered, in case

$$\frac{(1-\tau_{t+1} + \frac{\tau_{t+1}^2}{2})a_i}{1+r} \geq (1-\tau_t + \frac{\tau_t^2}{2})u_1 + \frac{(1-\tau_{t+1} + \frac{\tau_{t+1}^2}{2})u_2}{1+r}. \tag{12}$$

If the wage tax rate is expected to remain the same, that is $\tau_{t+1} = \tau_t$, this condition simplifies as

$$a_i \geq (1+r)u_1 + u_2.$$

As middle-aged individuals can save and borrow freely at the exogenous interest rate to smooth their consumption over their lifetime, the Fisher Separation Theorem applies. Optimal individual choices by a consumer can be characterized by a two-step optimization problem: one where individuals first decide whether to receive education if this is offered to them and then decide how to allocate their time between work in the official labor market and home production and a second one where, for a given expected lifetime income, individuals choose their utility-maximizing intertemporal consumption profile by borrowing and lending as middle-aged in the perfect capital market. Equations 12, 9, 10 and 11 provide a complete characterization of the consumers' choices for given levels of policy parameters.

## 5 Political equilibria

### 5.1 Voting with commitment

Political decisions are made by issue-by-issue direct democracy. Voting takes place simultaneously on the wage tax rate and on the provision of public education. This subsection analyzes voting when each generation assumes that the $\tau_t$ and $\widetilde{a}_t$ chosen will be in place over its whole lifetime. Therefore, there is commitment to maintaining the voting outcome for at least three periods. Assume that voting takes place once in three periods. The next subsection generalizes the results to the case in which voting takes place in each period.

When voting on the wage tax rate (the provision of education), citizens have to make a conjecture on the outcome of voting on the provision of education (the wage tax rate). With rational expectations, the outcome of the voting game validates these conjectures. Voting is costless and all citizens vote in each policy dimension. When voting with commitment, all citizens vote for their preferred alternative, while with repeated voting, citizens may vote strategically in a different way than they would vote with commitment to activate a trigger strategy to change the policy in the next period. Clearly, the two voting mechanisms may then lead to different outcomes. Assuming a simultaneous majority vote on the two issues generates a structure-induced equilibrium:[11]

**Definition 1** A *structure-induced equilibrium in voting* with commitment to maintaining the voting outcome in period $t$, also in periods $t+1$ and $t+2$, is, with a given provision of education in the previous period, $\widetilde{a}_{t-1}$, such a pair of $\widehat{a}_t$ and $\widehat{\tau}_t$ that:

(1) when citizens expect that the provision of education chosen will be $\widehat{a}_t$, $\widehat{\tau}_t$ is a Condorcet winner when voting on the wage tax rate
(2) when citizens expect that the wage tax rate chosen will be $\widehat{\tau}_t$, $\widehat{a}_t$ is a Condorcet winner when voting on the provision of education.

With commitment, $\widehat{a}_{t+2} = \widehat{a}_{t+1} = \widehat{a}_t$, $\widehat{\tau}_{t+2} = \widehat{\tau}_{t+1} = \widehat{\tau}_t$. Then future social security benefits are given generally by $b_{t+1} = b_{t+2}(\widetilde{a}_t, \widetilde{a}_t, \tau_t)$, and in equilibrium by[12]

$$b_{t+1} = b_{t+2}(\widehat{a}_t, \widehat{a}_t, \widehat{\tau}_t). \tag{13}$$

The same value of $\widetilde{a}_t$ which maximizes social security benefits with a given $\tau_t$ in the following period, maximizes social security benefits with the same $\tau_t$ in

[11] An alternative to simultaneous voting would be voting separately and sequentially over each dimension. In case of analyzing sequential voting, assumptions made on the timing of the choices may affect political outcome.

[12] A "hat" denotes the equilibrium outcome of a variable, while $\tau_t$ and $\widetilde{a}_t$ denote any possible values for the wage tax rate and the ability threshold above which citizens receive higher education. In period $t$, $\widehat{a}_t$ and $\widehat{\tau}_t$ refer to the expected equilibrium outcomes arising from the political process, while $\widetilde{a}_{t-1}$ is the threshold ability level that was in place in the previous period.

period $t+2$. For the old voting in period $t$, however, social security benefits are given by Eq. 8, taking in equilibrium value

$$b_t(\widetilde{a}_{t-1}, \widehat{a}_t, \widehat{\tau}_t) = R_t(\widetilde{a}_{t-1}, \widehat{a}_t, \widehat{\tau}_t) - (1 - F(\widehat{a}_t))q. \tag{14}$$

It is worth emphasizing the double role of individuals. As citizens, they vote for the $\tau$ and $\widetilde{a}$ they would prefer, given the conjecture they make on the equilibrium in voting in the other dimension. As consumer-producers, they take the voting outcome as given, choosing time allocation according to Eq. 9. The citizens vote to maximize the net present value of their current and future incomes. With perfect capital markets between the middle-age and the old-age, this maximizes their utility by Fisher's Separation Theorem. Denote the value of the sum of the current and future after-tax wage income, value of home production and transfers for a citizen of age $k$, $k$ being $y$ for the young, $m$ for the middle-aged and $o$ for the old, by $V^k$. For the young citizens, $V^y$ depends on the individual's own ability $a$, the provision of education $\widetilde{a}_t$ and the wage tax rate $\tau_t$. In case of becoming educated, $V_t^y$ is given by

$$V_t^y(a, \widetilde{a}_t, \tau_t) = \frac{(1 - \tau_t + \frac{\tau_t^2}{2})a}{1+r} + \frac{b_{t+2}(\widetilde{a}_t, \widetilde{a}_t, \tau_t)}{(1+r)^2}, \tag{15}$$

whereas, for those who remain uneducated, it is:

$$V_t^y(a, \widetilde{a}_t, \tau_t) = (1 - \tau_t + \frac{\tau_t^2}{2})u_1 + \frac{(1 - \tau_t + \frac{\tau_t^2}{2})u_2}{1+r} + \frac{b_{t+2}(\widetilde{a}_t, \widetilde{a}_t, \tau_t)}{(1+r)^2}. \tag{16}$$

The first argument in $V_t^y(a, \widetilde{a}_t, \tau_t)$ is the individual ability, the second argument is the ability threshold and the third argument is the wage tax rate. The preferred provision of education and wage tax rate for a young citizen with ability $a$ are given by[13]

$$\widetilde{a}_t^y(a, \widehat{\tau}_t) = \arg\max_{\widetilde{a}_t \in [0, \overline{a}]} V_t^y(a, \widetilde{a}_t, \widehat{\tau}_t) \tag{17}$$

$$\tau_t^y(a, \widehat{a}_t) = \arg\max_{\tau_t \in [0,1]} V_t^y(a, \widehat{a}_t, \tau_t). \tag{18}$$

These reaction functions specify the preferred choice in one dimension conditional on the equilibrium outcome in the other dimension.

[13] Note that if $q < ru_1$, then citizens with $a < (1+r)u_1 + u_2$ would not want to receive public education in case this would be offered to them. The model could be solved in this case by replacing $\widetilde{a}_t \in [0, \overline{a}]$ in Eqs. 17, 21 and 24 by $\widetilde{a}_t \in [(1+r)u_1 + u_2, \overline{a}]$. This would be without loss of generality as choosing $\widetilde{a}_t$ below $(1+r)u_1 + u_2$ would result in the same take-up of education on the part of potential students as choosing $\widetilde{a}_t = (1+r)u_1 + u_2$.

For the middle-aged, $V_t^m$ is given by

$$V_t^m(a, \widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) = (1 - \tau_t + \frac{\tau_t^2}{2})u_2 + \frac{b_{t+1}(\widetilde{a}_t, \widetilde{a}_t, \tau_t)}{1+r} \tag{19}$$

for those with $a < \widetilde{a}_{t-1}$ and

$$V_t^m(a, \widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) = (1 - \tau_t + \frac{\tau_t^2}{2})a + \frac{b_{t+1}(\widetilde{a}_t, \widetilde{a}_t, \tau_t)}{1+r} \tag{20}$$

for those with $a \geq \widetilde{a}_{t-1}$. Note that both the current ability threshold and the one from the previous period enter the value function for the middle-aged.

The preferred provision of education and wage tax rate for a middle-aged citizen with ability $a$ are given by

$$\widetilde{a}_t^m(a, \widetilde{a}_{t-1}, \widehat{\tau}_t) = \arg\max_{\widetilde{a}_t \in [0,\overline{a}]} V_t^m(a, \widetilde{a}_{t-1}, \widetilde{a}_t, \widehat{\tau}_t) \tag{21}$$

$$\tau_t^m(a, \widetilde{a}_{t-1}, \widehat{a}_t) = \arg\max_{\tau_t \in [0,1]} V_t^m(a, \widetilde{a}_{t-1}, \widehat{a}_t, \tau_t). \tag{22}$$

The presence of the term $\widetilde{a}_{t-1}$ captures the effect of past education policy on the current political preferences. Middle-aged citizens with $a < \widetilde{a}_{t-1}$ ($a \geq \widetilde{a}_{t-1}$) are uneducated (educated). Nonetheless, the optimal provision of education preferred by the middle-aged is, independent of whether they are educated or not, such that of solving $\arg\max_{\widetilde{a}_t \in [0,\overline{a}]} b_{t+1}$. This results as the only motivation for the middle-aged to support public education is to increase their future retirement benefits.

The value function for the elderly is given by

$$V_t^o(a, \widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) = R_t(\widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) - (1 - F(\widetilde{a}_t))q. \tag{23}$$

As in the case of the middle-aged, both the current ability threshold and the one in the previous period enter the value function of the elderly. Therefore, the preferred provision of education and wage tax rate for any elderly citizen are given by

$$\widetilde{a}_t^o(a, \widetilde{a}_{t-1}, \widehat{\tau}_t) = \arg\max_{\widetilde{a}_t \in [0,\overline{a}]} V_t^o(a, \widetilde{a}_{t-1}, \widetilde{a}_t, \tau_t) \tag{24}$$

$$\tau_t^o(a, \widetilde{a}_{t-1}, \widehat{a}_t) = \arg\max_{\tau_t \in [0,1]} V_t^o(a, \widetilde{a}_{t-1}, \widehat{a}_t, \tau_t). \tag{25}$$

The elderly oppose any provision of public education, thereby voting for $\widetilde{a}_t = \overline{a}$. They lose from public education both directly as the financing of public education competes for the same tax revenue as social security benefits, and indirectly as

more public education implies that a greater part of the young do not work. All the benefits from public education accrue only after the elderly have died. Even though the young have non-monotonic preferences concerning the provision of public education, it is possible to establish that:

**Lemma 1** *In any structure-induced equilibrium, the provision of education is chosen to maximize the future social security benefits, and is given by*

$$\widehat{a}_t = u_1 + u_2 + \frac{q}{\widehat{\tau}_t(1-\widehat{\tau}_t)}. \tag{26}$$

*All young citizens with* $a \geq \widehat{a}_t$ *strictly prefer to receive public education, rather than remain uneducated.*

*Proof* Proof is given in the Appendix.▪

Lemma 1 proves that the participation constraint for citizens having to be willing to receive higher education if it is offered them is never binding in an equilibrium.

To introduce the next notation

$$\omega_{t+1}(\widehat{a}_t) = F(\widehat{a}_t)u_1 + F(\widehat{a}_t)u_2 + \int_{a=\widehat{a}_t}^{\overline{a}} af(a)da. \tag{27}$$

Equation 27 gives the sum of the average productivity of the young and that of the middle-aged in the official labor market in periods $t+1$ and $t+2$, in case $\widetilde{a}_t = \widetilde{a}_{t+1} = \widehat{a}_t$. Introduce also the following notation: the tax rate preferred by a young citizen with ability $a$ who remains uneducated (becomes educated) is $\tau^y_{u,t}(a, \widehat{a}_t)$ ($\tau^y_{e,t}(a, \widehat{a}_t)$ ), and the tax rate preferred by the middle-aged citizen with ability $a$ who is uneducated (educated) is $\tau^m_{u,t}(a, \widetilde{a}_{t-1}, \widehat{a}_t)$ ($\tau^m_{e,t}(a, \widetilde{a}_{t-1}, \widehat{a}_t)$ ). The tax rate preferred by an old citizen is $\tau^o_t(a, \widetilde{a}_{t-1}, \widehat{a}_{t-1})$.

We can next derive

**Lemma 2** *Under conjecture* $\widetilde{a}_t = \widehat{a}_t$, *the preferred tax rates are given by*

$$\begin{aligned}
\tau^y_{u,t}(a, \widehat{a}_t) &= \max\left(0, \tfrac{\omega_{t+1}(\widehat{a}_t)-(1+r)^2u_1-(1+r)u_2}{2\omega_{t+1}(\widehat{a}_t)-(1+r)^2u_1-(1+r)u_2}\right) when \;\; a < \widehat{a}_t \\
\tau^y_{e,t}(a, \widehat{a}_t) &= \tfrac{\omega_{t+1}(\widehat{a}_t)-(1+r)a}{2\omega_{t+1}(\widehat{a}_t)-(1+r)a} \; when \;\; \widehat{a}_t \leq a < \tfrac{\omega_{t+1}(\widehat{a}_t)}{1+r} \\
\tau^y_{e,t}(a, \widehat{a}_t) &= 0 \;\; when \; a \geq \tfrac{\omega_{t+1}(\widehat{a}_t)}{1+r} \\
\tau^m_{u,t}(a, \widetilde{a}_{t-1}, \widehat{a}_t) &= \max\left(0, \tfrac{\omega_{t+1}(\widehat{a}_t)-(1+r)u_2}{2\omega_{t+1}(\widehat{a}_t)-(1+r)u_2}\right) when \;\; a < \widetilde{a}_{t-1} \\
\tau^m_{e,t}(a, \widetilde{a}_{t-1}, \widehat{a}_t) &= \tfrac{\omega_{t+1}(\widehat{a}_t)-(1+r)a}{2\omega_{t+1}(\widehat{a}_t)-(1+r)a} \; when \;\; \widetilde{a}_{t-1} \leq a < \tfrac{\omega_{t+1}(\widehat{a}_t)}{1+r} \\
\tau^m_{e,t}(a, \widetilde{a}_{t-1}, \widehat{a}_t) &= 0 \;\; when \;\; a \geq \tfrac{\omega_{t+1}(\widehat{a}_t)}{1+r} \\
\tau^o_t(a, \widetilde{a}_{t-1}, \widehat{a}_{t-1}) &= \tfrac{1}{2}.
\end{aligned}$$

*Proof* Proof is given in the Appendix.▪

The preferred tax rates of those who vote for a positive tax rate are increasing in $\omega_{t+1}(\widehat{a}_t)$ and decreasing in the interest rate $r$. We can next prove

**Lemma 3** *In any structure-induced equilibrium* $(\widehat{a}_t, \widehat{\tau}_t)$, *the preferred tax rates voted for by the different groups satisfy* $\tau^o_t(a, \widetilde{a}_{t-1}, \widehat{a}_{t-1}) > \tau^m_{u,t}(a, \widetilde{a}_{t-1}, \widehat{a}_t) \geq \tau^y_{u,t}(a, \widehat{a}_t) \geq \tau^y_{e,t}(\widehat{a}_t, \widehat{a}_t)$. *Furthermore,* $\tau^y_{e,t}(a, \widehat{a}_t) = \tau^m_{e,t}(a, \widetilde{a}_{t-1}, \widehat{a}_t) \quad \forall a$ *and* $\frac{\partial}{\partial a}\tau^y_{e,t}(a, \widehat{a}_t) < 0 \quad \forall a$ *with which* $\tau^y_{e,t}(a, \widehat{a}_t) > 0$.

*Proof* Follows by simple calculus from the expressions in Lemma 2.▪

Lemma 3 reports that the middle-aged uneducated prefer higher taxes than the young uneducated, and these prefer higher tax rate than any citizen who becomes educated. Those young yet-to-be-educated and those middle-aged who are already educated vote for the same wage tax rate, if they have the same abilities.

The existence of a structure-induced equilibrium with a voting with commitment does not guarantee that repeating voting in period $t+3$ would produce the same outcome. In other words, it is conceivable that some ability distributions would result in voting cycles. If voting in period $t+3$ would result in the same outcome as voting in period $t$, then the voting outcome in period $t$ is a steady-state outcome: once the economy is in it, repeated voting every three periods perpetuates this outcome. Define

**Definition 2** A combination of $(\widehat{a}_t, \widehat{\tau}_t)$ is a steady-state structure-induced equilibrium (SSSIE), if it is a structure-induced equilibrium when $\widetilde{a}_{t-1} = \widehat{a}_t$.

This definition is written for the initial condition $\widetilde{a}_{t-1}$, rather than for the voting outcome in period $t+3$, to highlight that a steady-state outcome arises if the ability threshold associated with this outcome is a fixed point in the political process. In other words, a steady-state structure-induced equilibrium is such that voting results in the same ability threshold as was the initial condition when the voting took place. Note that if voting outcome $(\widehat{a}_t, \widehat{\tau}_t)$ is a steady-state structure-induced equilibrium from period $t$ onwards, it is so also from period $t-1$ onwards.

In a SSSIE the outcome of the voting would not change even if the ballot were unexpectedly repeated already before the third period after the previous vote.[14] Whether there is a SSSIE depends on the distribution of $a$, as well as on $u_1$, $u_2$, $q$ and $r$. The possible SSSIEs are given by

**Proposition 1** *If* $(\widehat{a}_t, \widehat{\tau}_t)$ *is a steady-state structure-induced equilibrium, then the median voter on both* $a_t$ *and* $\tau_t$ *is, with* $\widetilde{a}_{t-1} = \widehat{a}_t$,

(1) *a middle-aged uneducated citizen if* $F(\widehat{a}_t) \geq \frac{1}{2}$;
(2) *a young citizen who remains uneducated if* $\frac{1}{4} \leq F(\widehat{a}_t) < \frac{1}{2}$;
(3) *a young citizen who becomes educated and a middle-aged educated citizen whose ability* $a_{iii}$ *is given by* $F(a_{iii}) = \frac{1}{4}$ *if* $F(\widehat{a}_t) < \frac{1}{4}$.

[14] Note that the policy conclusions do not rely on the assumption that voting would take place only once in every three periods. These are just simplifying steps to derive the results with repeated voting in the following subsection.

*Proof* Proof is given in the Appendix. ▪

Proposition 1 implies

**Corollary 1** *Even with three cohorts of the same size, the median voter may be a young citizen.*

Proposition 1 does not exclude a possibility of multiple equilibria. Whether there are multiple equilibria or not depends on the distribution of ability, and on $u_1$, $u_2$, $q$ and $r$:

**Proposition 2** *There may but need not be multiple equilibria.*

*Proof* Proof is given in the Appendix.▪

Proposition 2 demonstrates that a given ability distribution may support different equilibria, depending on initial conditions. Even when there are multiple equilibria, it is possible to establish a relative ranking of tax rates and public provision of education in these:

**Proposition 3** *If there are two steady-state structure-induced equilibria* $(\widehat{a}_t^1, \widehat{\tau}_t^1)$ *and* $(\widehat{a}_t^2, \widehat{\tau}_t^2)$*, then* $\widehat{\tau}_t^1 > \widehat{\tau}_t^2$ *implies* $\widehat{a}_t^1 < \widehat{a}_t^2$.

*Proof* By Lemma 2 and Lemma 3 and Proposition 1, $\widehat{\tau}_t^1 < \frac{1}{2}$ and $\widehat{\tau}_t^2 < \frac{1}{2}$. Then $\widehat{\tau}_t^1 > \widehat{\tau}_t^2$ implies that $\frac{1}{\widehat{\tau}_t^1(1-\widehat{\tau}_t^1)} < \frac{1}{\widehat{\tau}_t^2(1-\widehat{\tau}_t^2)}$, as $\frac{\partial}{\partial \tau}\tau(1-\tau) > 0$ when $0 < \tau < \frac{1}{2}$. By Eq. 26, this implies that $\widehat{a}_t^1 < \widehat{a}_t^2$. ▪

The result of Proposition 3 is intuitive: As public education receives majority support only when it increases the social security benefits, the number of students educated is increasing in the public stake in returns to education.

## 5.2 Repeated voting

This subsection analyzes an economy in which voting on $\widetilde{a}$ and $\tau$ takes place in every period. The equilibrium is now maintained by an implicit intergenerational contract supported by a suitable trigger strategy, instead of by commitment. Repeated voting without commitment generates a time-consistency problem in a sense that an individual voter could deviate, with repeated voting, from voting for his or her preferred choice in case with commitment with an attempt to activate a trigger strategy by such a deviation. Note that individual preferences change over a lifetime also when voting with commitment. This is not a time-inconsistency issue, and the political equilibrium specified does not rely on any citizen voting against his or her current preferences. Instead, the potential time-consistency problem is related to the commitment to a constant policy parameter in an implicit intergenerational contract, and the fact that with repeated voting, some citizens

may strategically deviate from voting for what would be their preferred choice in case of commitment.

Conde Ruiz and Galasso (2003) introduced the notion of a stationary subgame perfect structure induced equilibrium.[15] They applied subgame perfection to the notion of a structure-induced equilibrium introduced by Shepsle (1979). Agents vote according to a stationary subgame perfect strategy profile. A strategy for an individual is a mapping from the history of the voting outcomes to the wage tax rate $\tau_t$ and the provision of public education $\widetilde{a}_t$ that the individual votes for, and is subject to $\tau_t \in [0, 1]$ and $\widetilde{a}_t \in [0, \overline{a}]$. Let $s_t^k$ be the voting strategy profile over $\tau_t$ and $\widetilde{a}_t$ of all individuals belonging to generation $k$ in period $t$. Here $k$ can be either $y$ (young), $m$ (middle-aged) or $o$ (old). As the realized income of the middle-aged depends on the provision of education in the previous period, their voting strategy is denoted by $s_t^k(\widetilde{a}_{t-1})$ to make explicit that their voting strategy in period $t$ depends on the provision of education in period $t-1$. Denote the first period when the game is played and when the equilibrium strategy may be established, with the education of the middle-aged corresponding to the long-run equilibrium, by 0. The history of the game at period $t$, $h_t$, reports those combinations of $\widetilde{a}$ and $\tau$ chosen in all previous periods starting from $t = 0$: $h_t = \{(\widetilde{a}_0, \tau_0), (\widetilde{a}_1, \tau_1), ..., (\widetilde{a}_{t-2}, \tau_{t-2}), (\widetilde{a}_{t-1}, \tau_{t-1})\}$ when $t > 0$. The set of all possible past outcomes at time $t$ is denoted by $H_t$. The following formal definition is taken, with slight modifications, from Conde Ruiz and Galasso (2003):

**Definition 3** A voting strategy profile $s = \{(s_t^y \cup s_t^m \cup s_t^o)\}_{t=0}^{\infty}$ is a stationary subgame perfect structure-induced equilibrium (SSPSIE), if the following conditions are satisfied:

(1) $s$ is a subgame perfect equilibrium.
(2) At every period $t$, the equilibrium outcome associated with $s$ is a structure-induced equilibrium of the static game with commitment.
(3) In any period and for any history, $h_t \in H_t$, the sequence of equilibrium outcomes induced by $s$, is constant.

Conde Ruiz and Galasso (2003) assume a trigger strategy, specifying that the voting outcome converges to a complete abolition of the social security system in the case of the result in one period differing from the subgame perfect equilibrium with a once-and-for-all voting. In an intergenerational game with public provision of education, such a threat strategy would not always maintain the subgame perfect structure-induced equilibrium, resulting from voting with commitment, when there is repeated voting instead. The reason for this is that the high-ability young citizens with higher-than-average income would like to have wage taxation eliminated after they have completed their education.

With a threat strategy demanding a zero wage taxation after any deviation, the young, expecting to pay more in taxes than they receive back in social security benefits, may find it optimal to deviate. When voting with commitment, they

[15] Conde Ruiz and Galasso analyze simultaneous voting on the tax rate and early retirement provisions. They restrict the decision on early retirement provisions to a binary choice between full benefits or no benefits at all, whereas, this paper analyzes a two-dimensional policy space with a continuum of alternatives in both dimensions.

would prefer a lower wage tax rate than the median voter. By voting for a higher wage tax rate than the median voter, they would change the identity of the median voter in regime (3) of Proposition 1. In this way, they would cause an upward deviation in wage taxation during their youth when they do not pay taxes, triggering an elimination of public expenditures in the following period when they would otherwise pay taxes. Thus, the trigger strategy by Conde Ruiz and Galasso would not allow maintaining social security and public education with repeated voting. This results from the time-consistency problem that the high-ability young citizens face. However, there is a trigger strategy which allows the outcome of voting with commitment to be maintained also with repeated voting. This is:

**Definition 4** In the *implicit intergenerational contract voting strategy (IICVS)* related to an equilibrium $(\widehat{a}, \widehat{\tau})$, a citizen $i$, who prefers $\widetilde{a}_t^i(\widehat{\tau})$ and $\tau_t^i(\widehat{a})$ when voting with commitment, votes for this provision of education and wage tax rate in period $t = 0$, as well as in period $t > 0$, provided that the history for the previous periods satisfies $\widetilde{a}_j = \widehat{a} \quad \forall j \in \{0, 1, ..., t-1\}$ and $\tau_j \geq \widehat{\tau} \ \forall j \in \{0, 1, ..., t-1\}$. If these conditions are not satisfied, the citizen votes for the $\tau_t$ and $\widetilde{a}_t$ he or she preferred in period $t$, assuming that the (other) young and middle-aged citizens no longer vote for a positive wage tax rate due to the collapse of the implicit intergenerational contract. The individual-specific superscript $i$ captures both age and ability.

IICVS states that any other deviation from the intergenerational contract, except for a deviation to a higher wage tax rate, leads to the break-down of the intergenerational contract. Once the trust in the maintenance of the intergenerational transfer institutions is lost, they will collapse. If voting in some period would produce a higher wage tax rate than in the equilibrium, then the IICVS specifies that this does not violate the contract. Neither does it give a reason for changing the $\widetilde{a}$ and $\tau$ specified by the contract. Intuitively, working generations are not punished by an abolition of future social security if their voting would lead to paying higher social security benefits than specified by the contract.[16] This strategy supports a SSSIE also with repeated voting:

**Proposition 4** *Any steady-state structure-induced equilibrium, which would exist when voting with commitment, can be maintained also in repeated voting as a stationary subgame perfect structure-induced equilibrium.*

*Proof* Proof is given in the Appendix.▪

Proposition 4 shows that all results derived in the previous subsection for a steady-state structure-induced equilibrium hold also when voting is repeated in each period. Therefore, any implicit intergenerational contract on public education and social security benefits, which could be implemented with commitment, can be maintained also with repeated voting. A suitably formulated trigger strategy solves the time-consistency problem on the part of the high-income young citizens.

[16] An alternative trigger strategy would specify that all agents vote for $\widetilde{a} = \overline{a}$ and $\tau = 0$ if the contract has been violated in the previous period. While this would lead to the same outcome as the IICVS, voting for $\widetilde{a} = \overline{a}$ is weakly dominated by voting for any $\widetilde{a} \in [0, a^i]$ for the young with $a^i > (1+r)u_1 + u_2$. Likewise, for the elderly, voting for $\tau = 0$ is weakly dominated by voting for the rate of $\tau$, which maximizes the wage tax revenue for the social security benefits. With IICVS, no citizen votes for a weakly dominated strategy.

Repeated voting may sustain multiple equilibria, characterized in Proposition 1. As Proposition 3 reports, equilibria associated with higher tax rates are also associated with a wider provision of public education.

## 6 Conclusion

This paper has analyzed simultaneous voting on the wage tax rate and investment in public education when the young face borrowing constraints. The part of the tax revenue that is not used to finance public education is distributed to the elderly as social security benefits. When voting, citizens choose an optimal policy in each dimension, the wage tax rate and the provision of public education, subject to a conjecture of the outcome in the other dimension. This paper generalizes the results from previous literature, most notably Rangel (2003) and Boldrin and Montes (2002, 2005), by allowing productivity differences inside each cohort. The presence of productivity differences also implies that the median voter may be young, even when the three generations are of the same size.

Productivity differences introduce time-consistency problem in the presence of repeated voting, in case an implicit intergenerational contract is supported by a trigger strategy that punishes any deviations by a collapse of the implicit intergenerational contract. The high-productivity young citizens could strategically misrepresent their preferred tax rate in the first period by voting for a higher tax rate than the median voter, even if they would vote for a lower tax rate, if voting with commitment. They would do this to cause a deviation and activate the trigger strategy, rendering the implicit contract time-inconsistent. This can be avoided by a trigger strategy that punishes any deviations from the equilibrium public provision of education, and punishes also downward deviations from the equilibrium wage tax rate. A key mechanism to solve the time-consistency problem is that the implicit intergenerational contract specifies that upward deviations in the wage tax rate are not punished.

The basic model presented in this paper could be extended in several ways. Assuming a population growth rate would leave propositions 2, 3, and 4 unchanged, naturally, the population growth rate would change the ability thresholds, resulting in different equilibria in proposition 1. A more substantive change would be to introduce uncertainty on the lifetime or on realized income, or to allow parents to have altruistic motivations towards their children.

Another topic for further research would be to develop a computational general equilibrium model and use that to estimate the political equilibria that the model would predict in different countries. This would allow calculating a measure of the redistributive impact of the equilibrium policies, and also welfare comparisons between alternative steady-states. It would also allow testing the theory empirically. Running simple correlations between wage taxation and expenditures on public education as a share of GDP does not allow testing the theory, as it does not take into account country-specific differences in the productivity of labor or productivity distribution. Furthermore, it would be desirable to allow for a possibility that labor supplied by the educated and that supplied by the uneducated could be complements in production. These and other possible extensions are left for future research.

**Acknowledgements** I am indebted to Vesa Kanniainen, Katarina Keller, Tapio Palokangas, Jukka Pirttilä, Steinar Strøm, Andreas Wagener, conference participants, and three anonymous referees for useful comments. Financial support from the Yrjö Jahnsson Foundation is gratefully acknowledged.

## Appendix

*Proof of Lemma 1*

By Eqs. 7, 8, 15, and 17

$$\widetilde{a}_t^y(a,\widehat{\tau}_t) = u_1 + u_2 + \frac{q}{\widehat{\tau}_t(1-\widehat{\tau}_t)} \tag{28}$$

for those young with $a \geq \widetilde{a}_t^y(a,\widehat{\tau}_t)$ and for those with $a \leq u_1(1+r)+u_2$. Those young with the ability between these values vote either for this same ability threshold or for their own ability level as threshold, whichever would give them higher utility. The young never vote for a lower ability level than Eq. 28, as if they would, then they could increase the value of their old-age benefits by switching to this level. By Eqs. 7, 8, 19, 20 and 21, the preferred choice of all the middle-aged is given by

$$\widetilde{a}_t^m(a,\widehat{\tau}_t) = u_1 + u_2 + \frac{q}{\widehat{\tau}_t(1-\widehat{\tau}_t)}. \tag{29}$$

By Eq. 7, 8, 23 and 24, the elderly vote for $\widetilde{a}_t^o(a,\widehat{\tau}_t) = \overline{a}$. As the right-hand sides of Eqs. 28 and 29 are identical, and the elderly (who are one-third of the population) vote for a higher and none of the young for a lower cutoff level of ability, this is the median voter choice. Note that there is no single median voter; rather, the coalition consists of all the middle-aged and at least those young with $a \geq \widetilde{a}_t^y(a,\widehat{\tau}_t)$ or $a \leq u_1(1+r)+u_2$.

An individual with ability $a$ is willing to receive higher education, if and only if this results in higher after-tax income, that is

$$(1-\widehat{\tau}_t+\frac{\widehat{\tau}_t^2}{2})\frac{a}{1+r} \geq (1-\widehat{\tau}_t+\frac{\widehat{\tau}_t^2}{2})(u_1+\frac{u_2}{1+r}). \tag{30}$$

By assumption $q \geq ru_1$, any $a$ higher than (or equal to) $\widetilde{a}_t^y$ implied by Eq. 28 satisfies Eq. 30. Therefore, all citizens for whom public education is available prefer to receive it.

*Proof of Lemma 2* Insert Eq. 9 into Eq. 7 and then the resulting expression into Eq. 8. Then insert Eq. 27. By Eq. 13, $b_{t+1} = \quad b_{t+2} = \tau_t(1-\tau_t)\omega_{t+1} - (1-F(\widehat{a}_t))q$. Insert this into Eqs. 15, 16, 19 and 20. The solutions for $\tau_{u,t}^y(a,\widehat{a}_t)$ and $\tau_{e,t}^y(a,\widehat{a}_t)$ follow from the first-order conditions of Eq. 18, subject to the non-negativity constraint, and those for $\tau_{u,t}^m(a,\widehat{a}_{t-1})$ and $\tau_{e,t}^m(a,\widehat{a}_{t-1})$ follow, respectively, by the first-order conditions of Eq. 22. The solution for the elderly follows by maximizing Eq. 25, using Eqs. 7, 9 and 14.

*Proof of Proposition 1* As the provision of education chosen is given by Lemma 1 and is that preferred by the claimed median voter in each regime, it suffices to analyze voting on the wage tax rate. By Lemma 2, the middle-aged uneducated prefer higher taxes than the young uneducated, and these prefer higher taxes than anyone who is going to become educated. Furthermore, the preferred tax rate of the educated is declining in their ability and is the same whether they are young or middle-aged. With mass of each cohort normalized to unity, note that in scenario (1), the elderly and the middle-aged uneducated alone form more than half of the population. In scenario (2), the elderly and the middle-aged uneducated alone are not enough to form a majority. Together with the young uneducated, they form the majority. In scenario (3), the elderly and the uneducated are not alone in majority. As the young to-be-educated and the middle-aged educated prefer the same tax rate with the same ability, the median voter is a citizen with ability $a_{iii}$, whether young or middle-aged.

*Proof of Proposition 2* In each tentative equilibrium, the equilibrium tax rate can be solved from Lemma 2 and Proposition 1, given the assumption on the provision of education. This tentative equilibrium tax rate can then be inserted into Eq. 26 to verify whether the associated ability threshold is consistent with the initial assumption on who becomes educated. Assume first that 30 % of population have $a = 1.7$, 40 % have $a = 1.5$, and 30 % have $a = 0.8$, and that $u_1 = 0.1$, $u_2 = 0.8$, $q = 0.1$, and $r = 0.1$. There are three SSSIEs. In the first one, both citizens with ability level 1.7 and those with ability level 1.5 receive education, and $\tau_t = 0.215$. In the second one, only citizens with ability level 1.7 receive education, and $\tau_t = 0.186$. In the third equilibrium, no citizen receives education, and $\tau_t = 0.022$. In the first equilibrium, the median voter is a young citizen who remains uneducated, and in the second and third equilibria, a middle-aged uneducated. Assume next that 30 % of population have $a = 2$, 40 % have $a = 1.8$, and 30 % have $a = 0.8$, and that $u_1 = 0.5$, $u_2 = 0.5$, $q = 0.1$, and $r = 0.1$. Now the only equilibrium which exists is the one in which citizens with $a = 2$ and $a = 1.8$ become educated. Inserting the tentative equilibrium tax rate associated with the other equilibria into Eq. 26 shows that the associated ability threshold violates the initial assumption on who becomes educated.

*Proof of Proposition 4* Assume that $(\widehat{a}, \widehat{\tau})$ is a SSSIE with voting with commitment. It is sufficient to prove that with IICVS, the citizens either do not want to deviate or, if they would like to deviate, then their deviation does not change the outcome of the voting in a way that would result in the collapse of the intergenerational contract. The elderly have clearly no interest in deviating from voting for the $\tau$ and $\widetilde{a}$, which would maximize their current social security benefits. Neither do the middle-aged have any incentive to deviate from the $\tau$ and $\widetilde{a}$ they would prefer with commitment. A deviation downward in the wage tax rate or the provision of education would only result in them losing their social security benefits in the following period. The young uneducated, on the other hand, already vote for the $\widetilde{a}$ and $\tau$ that would maximize their lifetime utility, so they have no incentive to deviate. As for the young citizens who are going to become educated, they are, in any case, in minority when voting on the provision of education, so any deviation in that dimension by them would have no effect on the voting outcome. When voting on the wage tax rate, the young who are going to lose from income redistribution would prefer to have the wage taxation and

public provision of education abolished in future. However, they are already voting for a lower wage tax rate than the median voter, so that any deviation downward would not affect the outcome of the voting. The only way in which the young who prefer a lower wage tax rate to that preferred by the median voter can change the outcome of voting is by voting for a higher wage tax rate than that preferred by the median voter. By the definition of IICVS, a deviation upward would not cause the abolition of wage taxation and public provision of education.

Therefore, the young who will become educated cannot gain anything by deviating from voting for their preferred wage tax rate with voting with commitment. The threat point of the voting equilibrium $(\overline{a}, 0)$ following a punishable deviation is also a subgame perfect Nash-equilibrium. If the young and the middle-aged expect that social security benefits will not be maintained in the future, they have no interest in maintaining them after a deviation. This implies that the middle-aged would join the elderly in opposing any investment in public education. The elderly would still vote for $\tau > 0$ and the young, whose ability exceeds $(1 + r)u_1 + u_2$, for $\widetilde{a} < \overline{a}$, but both are in minority.

## References


Becker G, Murphy K (1988) The family and the state. J Law Econ 31(1):1–18

Bernheim BD, Nataraj S (2004) A solution concept for dynamic problems in political economy (mimeo), Occidental College

Boldrin M, Montes A (2002) The intergenerational state. Education and pensions (mimeo) University of Minnesota

Boldrin M, Montes A (2005) The intergenerational state. Education and pensions. Rev Econ Stud 72(3):651–664

Boldrin M, Rustichini A (2000) Political equilibria with social security. Rev Econ Dyn 3(1):41–78

Conde Ruiz JI, Galasso V (2003) Early retirement. Rev Econ Dyn 6(1):12–36

Cooley TF, Soares J (1999) A positive theory of social security based on reputation. J Polit Econ 107(1):135–160

Konrad KA (1995) Social security and strategic inter-vivos transfers of social capital. J Popul Econ 8(3):315–326

Kotlikoff L, Persson T, Svensson L (1988) Social contracts as assets: a possible solution to the time-consistency problem. Am Econ Rev 78(4):662–677

Miles D, Timmermann A (1999) Risk sharing and transition costs in the reform of pension systems in Europe. Econ Policy 14(29):251–286

Nataraj S (2004) Age bias in fiscal policy: why does the political process favor the elderly? (mimeo) Occidental College

OECD (2004a) Social expenditure database. SOCX. http://www.oecd.org/els/social/expenditure

OECD (2004b) Expenditure on educational institutions relative to gross domestic product, http://www.oecd.org/dataoecd/62/21/33670978.xls

OECD (2004c) Education at a glance. OECD Indicators, p 230

Pogue TF, Sgontz LG (1977) Social security and investment in human capital. Natl Tax J 30 (2):157–169

Rangel A (2003) Forward and backward intergenerational goods: why is social security good for the environment? Am Econ Rev 93(3):813–834

Shepsle KA (1979) Institutional arrangements and equilibrium in multidimensional voting models. Am J Polit Sci 23(1):27–59